Mass loss from a magnetically driven wind emitted by a disk orbiting a stellar mass black hole

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Abstract. The source of cosmic gamma–ray bursts (hereafter GRBs) is usually believed to be a stellar mass black hole accreting material from a thick disk. The mechanism for the production of a relativistic wind by such a system is still unknown. We investigate here one proposal where the disk energy is extracted by a magnetic field amplified to very large values $B \sim 10^{15}$ G. Using some very simple assumptions we compute the mass loss rate along magnetic field lines and then estimate the Lorentz factor Γ at infinity. We find that Γ can reach high values only if severe constraints on the field geometry and the conditions of energy injection are satisfied. We discuss the results in the context of different scenarios for GRBs.

I INTRODUCTION

Most of the sources which are now discussed to explain GRBs (the coalescence of two compact objects or the collapse of a massive star to a black hole (collapsar) [1–3]) lead to the same system : a stellar mass black hole surrounded by a thick debris torus. The release of energy by such a configuration can come from the accretion of disk material by the black hole or from the rotational energy of the black hole extracted by the Blandford-Znajek mechanism. The released energy is first injected into a relativistic wind and then converted into gamma–rays, via the formation of shocks probably within the wind itself [4,5]. The wind is finally decelerated by the external medium which leads to a shock responsible for the afterglow emission observed in the X–rays, optical and radio bands [6].

The production of the relativistic wind is a very complex question because of the very low baryonic load that has to be achieved in order to reach high values of the terminal Lorentz factor. Just a few ideas have been proposed and none appears to be fully conclusive. A first possibility to extract the energy from accretion is the annihilation of neutrino–antineutrino pairs emitted by the hot disk along the rotation axis of the system, which is a region strongly depleted in baryons due to centrifugal forces. The low efficiency of this process however requires high neutrino luminosities and therefore short accretion time scales [7]. Another possibility to extract the energy from accretion is to assume that the magnetic field in the disk is

amplified by differential rotation to very large values ($B \sim 10^{15}$ G). A magnetically driven wind could then be emitted from the disk with a fraction of the Poynting flux being eventually transferred to matter. The energy can also be extracted from the rotational energy of the black hole by the Blandford-Znajek mechanism [8].
We present here an exploratory study of the case where a magnetically driven wind is emitted by the disk. Matter is heated at the basis of the wind (by $\nu\bar{\nu}$ annihilation, viscous dissipation, magnetic reconnection, etc.) and then escapes, guided along the magnetic field lines. Section II describes a "toy model" to explore the behavior of such a wind. Despite its extreme simplicity, we expect that it can help to identify the key parameters controlling the baryonic load. Our results are presented in section III and discussed in section IV in the context of different scenarios for GRBs.

II A "TOY MODEL"

We solve the wind equations with the following simplifications : (i) we assume a geometrically thin disk and a poloidal magnetic field with the most simple geometry (straight lines making an angle θ with the disk) ; (ii) we consider that a stationnary regime has been reached by the wind; (iii) we use non–relativistic equations (to obtain the mass loss rate we just need to solve them up to the sonic point, where $v < 0.1c$) but we adopt the Paczyński-Wiita potential for the black hole

$$\Phi_{\rm BH} = -\frac{GM_{\rm BH}}{r - r_{\rm S}} \quad \text{with} \quad r_{\rm S} = \frac{2GM_{\rm BH}}{c^2} \ . \tag{1}$$

We write the flow equations (continuity, Euler and energy equations) in a frame corotating with the foot of the field line, anchored at a radius r_0 in the disk

$$\rho\ v\ s(x) = \dot{m}\ , \tag{2}$$

$$v\frac{dv}{dx} = g(x) r_0 - \frac{1}{\rho}\frac{dP}{dx}\ , \tag{3}$$

$$v\frac{d\epsilon}{dx} = \dot{q}(x) r_0 + v\frac{P}{\rho^2}\frac{d\rho}{dx}\ , \tag{4}$$

where $x = \ell/r_0$, ℓ being the distance along the magnetic field line, and ρ, P, ϵ and v are the density, pressure, specific internal energy and velocity in the flow.

The total acceleration $g(x)$ includes both gravitational and centrifugal terms. In this exploratory study the power deposited per unit mass $\dot{q}(x)$ only takes into account the heating and cooling due to neutrinos. We assume that the inner part of the disk is optically thick (which is probably justified for compact object mergers but is more questionnable for collapsars except for low α–viscosity ($\alpha < 0.01$) [10]). We include the following processes : neutrinos capture on free nucleons, neutrino scattering on relativistic electrons and positrons and neutrino–antineutrino

annihilation (heating); neutrino emission by nucleons and annihilation of electron–positrons pairs (cooling). The temperature distribution in the disk corresponds to a gemetrically thin, optically thick disk :

$$T_\nu(r) = T_* \left(\frac{r_*}{r}\right)^{3/4} \left(\frac{1-\sqrt{\frac{r_{in}}{r}}}{1-\sqrt{\frac{r_{in}}{r_*}}}\right)^{1/4} \quad (T_* \text{ is the temperature at } r_*) \; ; \tag{5}$$

The section of the wind $s(x)$ is easily related to the field geometry because the field and stream lines are coincident. We adopt the equation of state computed by [9] which includes nucleons, relativistic electrons and positrons and photons.

The acceleration $g(x)$ along a field line is negative up to $x = x_1$ for angles larger than $\theta_1 \simeq 60°$ ($60°$ is the exact value for a Newtonian instead of a Paczyński–Wiita black hole potential). For $x > x_1$, $g(x)$ is dominated by the centrifugal force. The sonic point of the flow is located at a distance x_s just below x_1 (the relative difference never exceeds 1%). We solve the flow equations in a classical way by inward integration along the field line. We start at the sonic point by fixing trial values of the temperature T_s and the density ρ_s from which we get the velocity v_s and the position x_s (from the condition of regularity at $x = x_s$) and then the value of the mass loss rate $\dot{m}$. We observe that at some position x_{cr}, the velocity v begins to fall off rapidly while T reaches a maximum $T_{max} \leq T_\nu(r_0)$. We adjust T_s and ρ_s so that x_{cr} is as close as possible to 0 and T_{max} to $T_\nu(r_0)$.

III RESULTS

We have studied the dependence of the mass loss rate $\dot{m}$ on the different model parameters and found the following expression :

$$\dot{m}(r) \sim 3.8\ 10^{13} \left(\frac{M_{\rm BH}}{2.5\ {\rm M}_\odot}\right) \left(\frac{T_\nu(r)}{2\ {\rm MeV}}\right)^{10} f\left[\frac{r}{r_{\rm g}};\theta(r)\right]\ {\rm g/cm^2/s}\ . \tag{6}$$

The geometrical function f is normelized in such a way that it is equal to unity for $r = 4\ r_{\rm g}$ and $\theta(r) = 85°$. The very strong dependance of $\dot{m}$ with $T_\nu(r)$ (tenth power) is in agreement with what is found for neutrino driven winds in spherical geometry [11]. Figure 1 shows that $\dot{m}$ also strongly depends on the inclination angle. The other important parameters are the position in the disk and the mass of the black hole, while $\dot{m}$ depends only weakly on all other parameters like the size of the optically thick region (here $r_{\rm in} = 3\ r_{\rm g}$ and $r_{\rm out} = 10\ r_{\rm g}$). In the more general case where the source of heating is not restricted to neutrino processes but can also include viscous dissipation, magnetic reconnection, etc, we have obtained a very simple and general analytical approximation for $\dot{m}$ [12]

$$\dot{m} \sim \frac{\dot{e}}{\Delta\Phi}\delta\ , \tag{7}$$

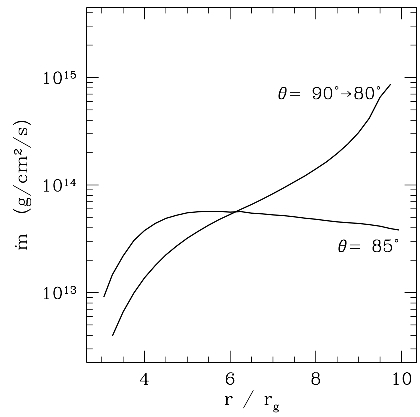

FIGURE 1. Mass loss rate from the disk for a constant ($\theta = 85°$) and decreasing (from 90° to 80° between 3 and 10 $r_{\rm g}$) inclination of the field lines. The disk temperature is $T_* = 2$ MeV at $r_* = 4\ r_{\rm g}$. The mass of the black hole is $M_{\rm BH} = 2.5\ {\rm M}_\odot$.

where $\dot{e}$ is the rate of energy deposition (in erg/cm^2/s) between the plane of the disk ($x = 0$) and the sonic point ($x = x_s \simeq x_1$), $\Delta\Phi$ is the difference of potential (gravitational+centrifugal) between $x = 0$ and $x = x_1$ and δ is a factor close to unity depending on the distribution of energy injection between $x = 0$ and $x = x_s$.

We can now estimate the average Lorentz factor $\bar{\Gamma} = \dot{E}/\dot{M}c^2$ at infinity. The total mass loss rate $\dot{M}$ and the power injected into the wind $\dot{E}$ are given by

$$\dot{M} = 2\int_{r_{in}}^{r_{out}} \dot{m} 2\pi r dr = 2.6\ 10^{26} \left(\frac{M_{\rm BH}}{2.5\ {\rm M}_\odot}\right)^3 \left(\frac{T_*}{2\ {\rm MeV}}\right)^{10}\ F_{\rm geo}\ {\rm g/s} \tag{8}$$

$$\text{and } \dot{E} = 2\ 10^{51} \left(\frac{\Omega/4\pi}{0.1}\right) \left(\frac{f_\gamma}{0.05}\right)^{-1} \left(\frac{\dot{E}_\gamma}{10^{51}/4\pi\ {\rm erg/s/sr}}\right)\ {\rm erg/s}\ , \tag{9}$$

where $F_{\rm geo} = \int_{r_{in}/r_{\rm g}}^{r_{out}/r_{\rm g}} f\left[x;\theta(x)\right] x dx$ is a function of the field geometry only; $\dot{E}_\gamma$ is the burst power in gamma–rays, $\Omega/4\pi$ is the beaming factor and f_γ is the efficiency for the conversion of kinetic energy into gamma–rays. The wind is powered by accretion but at the same time the disk is heated by viscous dissipation and cools by emitting neutrinos. We assume that these losses represent a fraction α of the power $\dot{E}$ injected into the wind, so that we can estimate T_* at $r_* = 4\ r_{\rm g}$:

$$\dot{E}_\nu = \alpha \dot{E} = 2\int_{r_{in}}^{r_{out}} \frac{7}{8}\sigma T_\nu^4(r)\ 2\pi r dr \tag{10}$$

$$\text{and } T_* = 1.72\ \alpha^{\frac{1}{4}}\ \left(\frac{M_{\rm BH}}{2.5\ {\rm M_{odot}}}\right)^{-\frac{1}{2}} \left(\frac{\Omega/4\pi}{0.1}\right)^{\frac{1}{4}} \left(\frac{f_\gamma}{0.05}\right)^{-\frac{1}{4}} \left(\frac{\dot{E}_\gamma}{10^{51}/4\pi\ {\rm erg/s/sr}}\right)\ {\rm MeV}\ . \tag{11}$$

From equations (8), (9) and (11), we can calculate the average Lorentz factor

$$\bar{\Gamma} = \frac{8500}{F_{\rm geo}}\alpha^{-\frac{5}{2}} \left(\frac{M_{\rm BH}}{2.5\ {\rm M}_\odot}\right)^2 \left(\frac{\dot{E}_\gamma}{10^{51}/4\pi\ {\rm erg/s/sr}}\right)^{-\frac{3}{2}} \left(\frac{\Omega/4\pi}{0.1}\right)^{-\frac{3}{2}} \left(\frac{f_\gamma}{0.05}\right)^{\frac{3}{2}}\ . \tag{12}$$

The value of $F_{\rm geo}$ is 56 for a constant inclination $\theta = 85°$ and 250 if θ decreases from 90° to 80° between $r = 3$ and 10 $r_{\rm g}$. We therefore conclude that large terminal Lorentz factors can be reached only if several severe constraints are satisfied : (i) low $F_{\rm geo}$ values, i.e. quasi–vertical field lines; (ii) low α values, i.e. good efficiency for energy injection into the wind with little dissipation ; (iii) low value of $\Omega/4\pi$, i.e. necessity of beaming. With the more general equation (7) we can obtain another simple and useful constraint : if the power $\dot{e}$ deposited below the sonic point represents a fraction χ of the total power $\dot{e}_{\rm tot}$ injected into the wind, we have

$$\Gamma \sim \frac{\dot{e}_{\rm tot}}{\dot{m}c^2} \sim \frac{\Delta\Phi/c^2}{\delta\chi} \ . \qquad (13)$$

For $r = 4$ $r_{\rm g}$ and $\theta = 85°$, we obtain $x_1 = 2.182$ and $\Delta\Phi/c^2 = 0.18$ which implies that χ should not exceed 10^{-3} to have $\Gamma > 100$!

IV DISCUSSION

This study is clearly limited by its crude assumptions. However the severe constraints we get show how difficult it may be to produce a relativistic MHD wind from the disk. An optimistic view of our results would be to consider that this difficulty could just be a way to explain the apparent discrepancy between the observed rate of GRBs and the birthrate of sources in the collapsar scenario, most collapsars failing to give a GRB. A more pessimistic point of view would be to conclude that the baryonic load of such winds is never sufficiently low so that they remain non relativistic. If one choose to rely on the Blandford-Znajek mechanism to power the wind [8] it should however be checked that this process is not "contaminated" by frozen material carried along magnetic field lines coming from the disk and trapped by the black hole.

REFERENCES

1. Narayan, R., Paczyński, B. and Piran, T., *ApJ*, **395**, L83 (1992).
2. Woosley, S.E., *ApJ*, **405**, 273 (1993).
3. Paczyński, B., *ApJ*, **494**, L219 (1998).
4. Rees, M.J. and Mészáros, P., *ApJ*, **430**, L93 (1994).
5. Daigne, F. and Mochkovitch, R., *MNRAS*, **296**, 275 (1998).
6. Wijers, R.A.M.J, Rees, M.J. and Mészáros, P., *MNRAS*, **288**, L51 (1997).
7. Ruffert, M., Janka, H.T., Takahashi, K. and Schaeffer, G., *A& A*, **319**, 122 (1997).
8. Lee, H.K., Wijers, R.A.M.J. and Brown, G.E., *ASTRO–PH 9906213* (1999).
9. Bethe, H.A., Applegate, J.H. and Brown, G.E., *ApJ*, **241**, 343 (1980).
10. Popham, R., Woosley, S.E. and Fryer, C., *ApJ*, **518**, 356 (1999).
11. Duncan, R.C., Shapiro, S.L. and Wasserman, I., *ApJ*, **309**, 141 (1986).
12. Daigne, F. and Mochkovitch, R., *in preparation.*